Statistical production of antikaon nuclear bound states in heavy ion collisions

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Abstract

Recently it was conjectured that the strongly attractive antikaon–nucleon potential can result in the formation of antikaon nuclear bound states. We discuss the formation of such states as possible residues in heavy ion collisions. In this context, we calculate the excitation functions of single- and double-K^- clusters in terms of the statistical thermal model. We show that, if such objects indeed exist, then, in heavy ion collisions, the single-K^- clusters are most abundantly produced at present SIS energies, while the double-K^- clusters show a pronounced maximum yield in the energy domain of the future accelerator at GSI. This is a direct consequence of: i) the baryonic dominance in low energy heavy ion collisions and the large baryonic content of the antikaonic bound states; ii) the strong energy dependence of strangeness production at low energies. The production yields of double-strange clusters is compared with that of double strange baryons. It is shown that at SIS energy there is a linear scaling relation of the Ξ^-/K^+ with K^+/p yields ratio.

Key words: bound kaonic clusters, statistical model
PACS: 25.75.-q, 25.75.Dw, 24.10.Pa

1 Introduction

Heavy ion collisions provide an experimental environment to study the properties of nuclear matter under conditions of high energy density well above its nuclear saturation. The observed particle production yields and spectra were shown to be sensitive to the collective and possible medium effects in a thermal system created in heavy ion collisions [1,2,3,4,5,6]. In low energy collisions at SIS accelerator a particular role has been attributed to strange mesons production. There are theoretical expectations that kaons and antikaons should experience different interactions in high density nuclear matter [4,7,8,9,10,11,12].

Results obtained within a coupled-channel approach based on the chiral SU(3) effective Lagrangian show [11,12] that K^+ undergoes repulsive interaction whereas K^- undergoes attractive interaction with nucleons. However, it is not established yet how strong is the antikaon attraction with the nuclear matter. The earlier study [13,14] of kaonic atoms has indicated a very strong attraction (150–200 MeV). More recent results [15,16] suggest relatively shallow (50–60 MeV) potential. Also, self consistent calculations [17] of in–medium potential for $\bar{K}$ that include its in-medium mass-shift find a shallow antikaon optical potential not exceeding 30 MeV. The effects of higher partial waves [18] and the pion dressing [19] are also modifying the strength of in-medium antikaon interactions. The self-consistent coupled channel calculations performed so far suggest that at normal nuclear matter density the in-medium potential for antikaons is at most 80 MeV.

The attractive in-medium optical potential for antikaons and repulsive interaction of kaons should, in general, result in a distinct difference between K^- and K^+ production cross section and flow pattern in heavy ion collisions. Indeed, the recent results of the KaoS [20] and FOPI [21] collaborations indicate such a difference. Within the context of dynamical transport [22,23,24,25] and statistical model [26,27] calculations of strangeness production at SIS energies it was argued that the observed properties of kaon and antikaon production can be to a large extent understood if an in medium kaon–nucleon potential is taken into account.

It was realized recently that, in spite of the strong absorptivity of the K$^-$-nucleus potential, sufficiently narrow deeply bound kaonic atom states could exist irrespective of how attractive the K$^-$-nucleus potential is [28]. In addition, the topic of strangeness at low energies has gained an additional facet. Based on phenomenologically constructed K^--nucleon interactions, it was argued that an additional consequence of a strongly attractive K^--nucleon potential could be the formation of deeply bound nuclear K^- states [29,30,31]. Such states should exhibit central nuclear densities which exceed by 4–9 times the normal nuclear density. They are also characterized by large binding energies ($E_K \simeq 100$ MeV) [1] and widths of 13–40 MeV [29,30,32]. In Table 1 we summarize the predicted K^- clusters and their basic physical properties [31].

There are already some experimental indications [30,33] that K^- nuclei could indeed be formed. An experiment carried out at KEK has found [33] a peak in a neutron missing momentum spectrum from the ^4He(stopped K^-, n) reaction, indicating a candidate for the predicted bound state of $ppnK^-$. More recently a similar signal has been observed [33] in a proton missing spectrum indicating a narrow ($I = 1$) state with a mass of 3117 MeV which could correspond to $pnnK^-$ cluster.

The K^- nuclei, if they exist, could be not only populated by direct reactions but could appear in any system if the K^- is immersed in high density nuclear environment. The dense medium produced in heavy ion collisions could be a favorable environment for the formation of such nuclei [31]. Thus, the K^- clusters may be possibly found as residues of

[1] This value is larger than the upper limit expected in self consistent coupled channel calculations.

Table 1
The properties of the predicted K^- clusters [31]: isospin (I, I_z), spin-parity (J^π), mass (Mc^2), binding energy (E_K), width (Γ_K), nuclear density in the center of the system $(\rho(0))$ and radius $(R_{\rm rms})$.

K^- – cluster	(I, I_z)	J^π	Mc^2 [MeV]	E_K [MeV]	Γ_K [MeV]	$\rho(0)$ [fm^{-3}]	$R_{\rm rms}$ [fm]
pK^- ($\Lambda(1405)$)	$(0, 0)$	$(1/2)^-$	1407	27	40	0.59	0.45
ppK^-	$(1/2, 1/2)$	0^-	2322	48	61	0.52	0.99
$pppK^-$	$(1, 1)$	$(3/2)^+$	3122	186	13	1.56	0.81
$ppnK^-$	$(0, 0)$	$(1/2)^-$	3152	170	21	1.50	0.72
$ppnK^-$	$(1, 0)$	$(3/2)^+$	3118	190	13	1.56	0.81
$pnnK^-$	$(1, -1)$	$(3/2)^+$	3117	191	13	1.56	0.81
$ppppK^-$	$(3/2, 3/2)$	0^-	4171- 90	75+90	20	1.68	0.95
$pppnK^-$	$(3/2, 1/2)$	0^-	4135-90	113+90	20	1.29	0.97
$ppnnK^-$	$(3/2, -1/2)$	0^-	4135-90	114+90	20		1.12
ppK^-K^-	$(0, 0)$	0^+	2747	117	35		
$pppK^-K^-$	$(1/2, 1/2)$	$(3/2)^+$	3580-180	220+180			
$ppnK^-K^-$	$(1/2, -1/2)$	$(3/2)^+$	3582-180	221+180	37	2.97	0.69
$pppnK^-K^-$	$(1, 0)$	0^+	4511-180	230+180	61	2.33	0.73

relativistic heavy ion collisions. This is particularly the case, as will be also argued in this paper, for the SIS energy range where antikaons are produced in the high baryon density environment. Furthermore the temperature reached at SIS energy is of the order or lower than the expected binding energy of K^-–nuclear clusters. Consequently, if such K^- states are produced in heavy ion collisions they cannot easily be destroyed by rescattering with the surrounding thermal medium.

On the other hand it was argued [18,34] that, in the heavy ion environment, the structure of the antikaonic potential could be modified. In heavy ion collisions antikaons are produced with finite momentum, thus formation of K^-–nuclear clusters is determined by the strength of the potential at finite, instead of vanishing momenta. Theoretical studies [18,34] showed a non-trivial momentum dependence of the potential which influences both its real and imaginary part. There is in general a decrease of the strength of the antikaon interaction in nuclear matter at finite momentum.

At high density [35] and/or temperature the spectral function of antikaons is theoretically expected to be so strongly modified that the antikaon is not anymore a quasi-particle but rather a broad state with temperature and/or density-dependent width. In such a case it is rather difficult to consider the binding of this broad state with the surrounding nucleons to form a bound nuclear cluster. In ref. [34] it was even argued that the effect of scattering

of antikaons with finite momentum in a hot medium can wash out the attractive nature of the potential. If this is indeed the case, then the production of K^-–nuclear clusters could be suppressed in heavy ion collisions or their formation restricted to the final stage, close to freezeout. We have to stress, however, that a detailed understanding of in-medium antikaon properties is at present not available.

In this paper we assume that kaonic bound states can be produced in heavy ion collisions and discuss the production yields of single- and double-K^- clusters at chemical freezeout in terms of the statistical model. We compare the production probabilities of such objects relative to those expected for Λ or Ξ^- strange particles in terms of the collision energy and system size dependence. [2]

2 Outline of the statistical model

The statistical model has been shown to be well suitable to describe different particle and light nuclear cluster yields [1,36,37,38,39] obtained in heavy ion collisions in a broad energy range from SIS, through, AGS, SPS and RHIC. To formulate the model we use the statistical operator of hadron resonance gas HRG [1]. The resonance contribution is of crucial importance to describe effectively the strong interactions of produced hadrons in the vicinity of the chemical freezeout. On the other hand, resonances provide an essential contribution to light particle yields. In the standard formulation all resonances with well established decay properties are included in the partition function [1,36,37].

In nucleus–nucleus collisions particle production is constrained by the conservation laws. Thus, modelling the partition function one needs to implement the conservation of the baryon number, electric charge and strangeness. The first two conservation laws are usually included on the grand canonical level and are controlled in the statistical operator through the corresponding chemical potentials. Strangeness conservation must be, however, introduced exactly within the canonical ensemble [41] where it is not anymore controlled by the strange chemical potential or corresponding fugacity parameter. This is particularly the case if one considers strangeness production in heavy ion collisions at SIS energy [42]. There, strange particles and antiparticles are very rarely produced and are strongly correlated in order to preserve strangeness conservation. Consequently, the thermal phase space available for strangeness production is suppressed [43,44]. This suppression is effectively described by the exact strangeness conservation through canonical formulation of the partition function. A detailed description of particle production in terms of the statistical model can be found in Ref. [1,46]. In the following we summarize the basic results which are required to quantify the yields of strange K^- – clusters and also other strange particles produced in heavy ion collisions. We emphasize that no medium modifications of kaon properties are considered within the present model.

[2] The present calculations superseed the preliminary ones shown in ref. [45], for which an incorrect assignment of the quantum numbers was done.

The canonical partition function of a hadron resonance gas in a thermal system with total strangeness S is obtained [46] as:

$$Z^C_{S=0} = e^{S_0} \sum_{n=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} a_3^p a_2^n a_1^{-2n-3p} I_n(x_2) I_p(x_3) I_{-2n-3p}(x_1), \tag{1}$$

where $a_i = \sqrt{S_i/S_{-i}}$, $x_i = 2V\sqrt{S_i S_{-i}}$ and S_i is the sum of all Z^1_k partition functions

$$Z^1_k = \frac{g_k}{2\pi^2} m_k^2 T \, K_2(m_k/T) \, \exp(B_k\mu_B + Q_k\mu_Q) \tag{2}$$

for particle species k carrying strangeness $S_k = i$, the baryon number B_k and electric charge Q_k. The $I_n(x)$ in (1) are the modified Bessel functions.

The density n^s_k of particle k having the strangeness s is obtained from Eq. (1) as:

$$n^s_k = \frac{Z^1_k}{Z^C_{S=0}} \sum_{n=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} a_3^p a_2^n a_1^{-2n-3p-s} I_n(x_2) I_p(x_3) I_{-2n-3p-s}(x_1). \tag{3}$$

In the canonical formulation of strangeness conservation the density of strange particles is explicitly volume dependent through the arguments x_i of the Bessel functions in Eq. (3). In the application of Eq. (3) to the description of particle production in heavy ion collisions this volume parameter was interpreted as the strangeness correlation volume which depends on the number of participants [44]. For large V and for high enough temperature such that all $x_i >> 1$ the canonical result (3) is converging to its GC value where strangeness conservation is controlled by the corresponding chemical potential [1]. Obviously, in the GC limit particle densities are not any more dependent on the volume parameter. In heavy ion collisions the GC approximation was found to be adequate for energies beyond AGS [1]. For lower collision energies, in particular for SIS, the suppression due to canonical effects can even exceed an order of magnitude for the yields of $S = \pm 1$ strange particles. The antikaonic clusters are strangeness $S = -1$ or $S = -2$ objects, thus in the context of a thermal model their production yields is to be suppressed due to the exact strangeness conservation constraints.

Note that the above formulations do not explicitely include finite widths. However, in the numerical realization of the model, all widths of resonances and of the kaonic bound states are included following the methods described in ref. [1].

3 Statistical production of K^- nuclei in heavy ion collisions

In the thermal model the strange particle density (3) depends in general on four independent parameters: the temperature, the charge and baryon chemical potential and the correlation volume. Two of these parameters are fixed through the initial conditions. In A–A collisions the isospin asymmetry in the initial state fixes the charge chemical potential whereas the correlation volume parameter V is taken to scale with A [42,44] (see the discussion in Section 3.2). Thus, only the temperature T and the baryon chemical potential μ_B are left as independent parameters. In heavy ion collision the temperature and baryon chemical potential are the parameters which characterize the properties of the collisions fireball at chemical freezeout. These parameters are specific to a given collision energy and can also vary with the number of participating nucleons in the collision.

3.1 Energy dependence

To quantify the production yields of K^- clusters in heavy ion collisions for different collision energies we adopt here the $\sqrt{s}_{NN}$ dependence of T and μ_B following the phenomenological chemical freezeout conditions of fixed energy per particle, $< E > / < N >= 1$ GeV [48]. This condition was shown to be consistent with chemical particles freezeout in central Au+Au or Pb+Pb collisions obtained in the energy range from SIS up to RHIC. The thermal parameters extracted from the midrapidity data and for energies beyond the top AGS are also well consistent with the fixed net baryon density as a chemical freezeout condition [39].

In nucleus–nucleus collisions the particle yield rather than its density is an observable. In the thermal model the strange particle yield is obtained from Eq. (3) when multiplying the result by the fireball volume at the chemical freezeout. As the fireball volume is not a directly measurable quantity [3], to avoid an extra parameter we normalize the K^- cluster yield to the Λ yield. In this case the ratio depends only on the chemical freezeout values of T and μ_B.

In Fig. 1 we show the resulting excitation function for $S = -1$ and $S = -2$ antikaonic clusters calculated in the thermal model described above. The required parameters characterizing the physical properties of these objects were taken from Table 1. In Fig. 1 the K^-–cluster/Λ ratio is compared to the ratio Ξ^-/Λ. For $S = -1$ clusters the yield/Λ is seen in Fig. 1 to be a decreasing function of $\sqrt{s}$. This is due to an interplay between T and μ_B variation with the collision energy. The increase of T with $\sqrt{s}$ should in general enhance the yield/Λ ratio. However, due to the simultaneous drop in μ_B the yield/Λ ratio decreases with $\sqrt{s}$. The largest production rate of $S = -1$ and $B > 1$ clusters is expected at SIS energy due to the large $\mu_B \sim 0.7 - 0.82$ GeV reached at the chemical freezeout. For $B = 1$ state a drop in temperature at SIS energy is not anymore compensated by an

[3] The fireball volume can be infered from HBT studies [40].

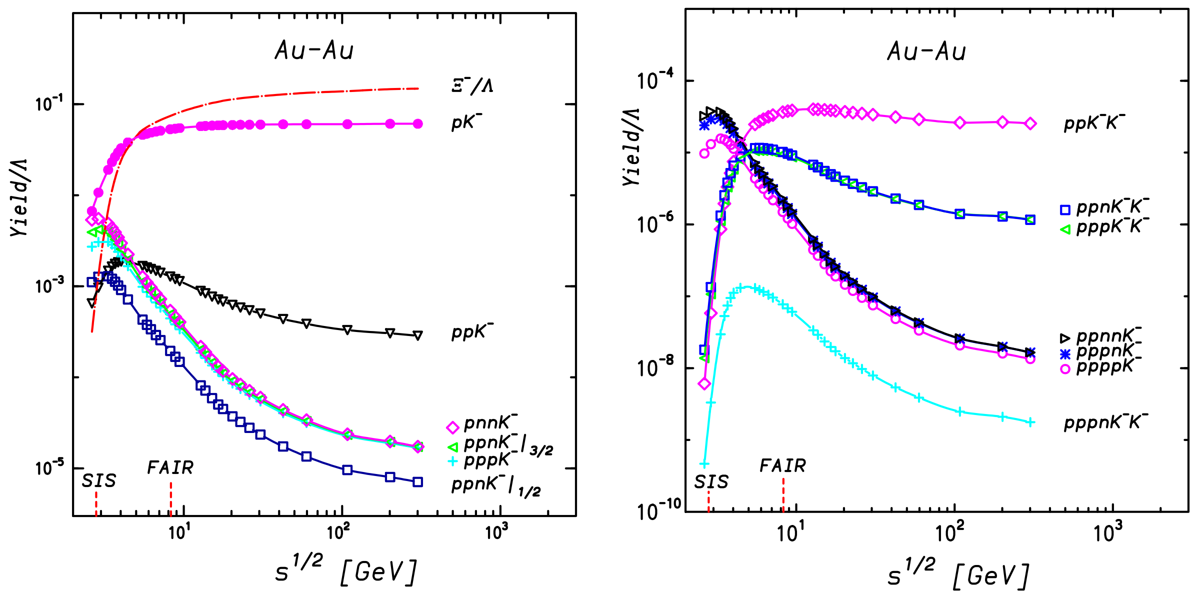

Fig. 1. Excitation function of the yields of antikaon bound states relative to Λ yields. The calculations are done along the freezeout curve of fixed $E/N = 1$ GeV [48]. For comparison, also show in the left panel is the Ξ^-/Λ yield ratio. The vertical lines indicate the upper energies accessible in experiments at SIS and FAIR.

increase in μ_B, resulting in a depletion of the yield/Λ ratio. There is also a quite steep decrease of the $S = -2$ cluster yields towards SIS energy as seen in Fig. 1. This is due to: i) the large cluster mass, which through the Boltzmann factor in Eq. (2) reduces the thermal particle phase space; ii) the strangeness suppression effect. The strangeness suppression factor is common for all particles carrying the same strange quantum numbers, thus it is cancelled out in the yield/Λ ratio for $S = -1$ clusters. For $S = -2$ states this is not any more the case as the strangeness suppression is increasing with the strangeness content of the particle [44].

At SIS energy the yield of $S = -1$ low lying kaonic clusters is larger than Ξ^- yield. This is due to the large baryon number of these objects which through the chemical potential increases the thermal phase space of K^- nuclei beyond that accessible for Ξ^-. There is also an additional suppression of Ξ^- due to the strangeness conservation as discussed above. For heavier $S = -1$ clusters and for $S = -2$ states (see Fig. 1) the production probability is already an order of magnitude lower than for Ξ^-.

The excitation functions for antikaonic bound states shown in Fig. 1 were obtained for central Au+Au collisions. Changing the colliding system is differently affecting the $S = -1$ and $S = -2$ yields. For $S = -1$ clusters the Yield/Λ ratio is essentially independent on the number of participants. This is due to the cancellation of the correlation volume dependent factors in the above yields ratio. Some changes in $S = -1$ Yield/Λ ratio with A in A–A collisions are to be expected due to the different isospin asymmetry in the initial state. The isospin effect appears e.g. in Fig. 1 as the difference between $pppK^-$ and $pnnK^-$ yield at low $\sqrt{s}$.

The results of Fig. 1 indicate clearly that the SIS energy range is the most appropriate to search for single-K^- clusters in heavy ion collisions. Since such an experimental program is underway at the SIS accelerator [49], we further focus on the model predictions in this energy range. On the other hand, the yield of double-K^- clusters is maximal in the energy range of the future accelerator planned at GSI [50], where it can be addressed within the CBM experiment [51].

3.2 Kaonic nuclear clusters at SIS energy

The freezeout temperature in A–A collisions at SIS energies is so low ($T \simeq 50 - 70$MeV) that all arguments x_i in (3) are less than unity. In the limit of $x_i << 1$ it is sufficient to take only the term with $n = p = 0$ in Eq. (3). In addition, expanding the Bessel functions $I_i(x)$ for $x_i \to 0$ it is also sufficient to consider only the leading term. Within the above approximations the density n_i^s of particle i carrying strangeness s is obtained from Eq. (3) as

$$n_i^s \simeq Z_i^1 \frac{V^{|s|}}{|s|!} \times \begin{cases} (S_{-1})^{|s|} \ , \ s > 0 \\ (S_{+1})^{|s|} \ , \ s < 0 \end{cases} \tag{4}$$

where

$$S_{-1} \simeq Z_\Lambda^1 + Z_{\Sigma^0}^1 + Z_{\Sigma^+}^1 + Z_{\Sigma^-}^1 \quad \text{and} \quad S_{+1} \simeq Z_{K^+}^1 + Z_{K^0}^1 \tag{5}$$

with Z_i^1 defined as in Eq. (2).

Considering the structure of Eqs. (4) and (5) it is clear that strange particles appear in pairs to guarantee the total strangeness to be exactly zero. In A–A collisions at SIS energy the correlation volume parameter V is supposed to scale with A as $V \simeq A\,V_0$ with $V_0 = (4/3)\pi r_0^3$ with $r_0 \simeq 1.1$ fm. Such parametrization of the correlation volume together with the canonical description of strangeness production is consistent with Au–Au and Ni–Ni data on K^+ and K^- production at SIS energies [42,52].

The $\bar{K}$ nuclear clusters, if produced during heavy ion collisions, should follow the thermal model systematics. The cluster yields $\langle N\rangle_{S=-1}^c$ and $\langle N\rangle_{S=-2}^c$ carrying strangeness $S = -1$ and $S = -2$ respectively and normalized to the number of Λ are found from Eq. (4) as

$$\frac{\langle N\rangle_{S=-1}^c}{\langle \Lambda \rangle} \simeq \frac{Z_{S=-1}^1}{Z_\Lambda^1 + Z_{\Sigma_0}^1} \ , \qquad \frac{\langle N\rangle_{S=-2}^c}{\langle \Lambda \rangle} \simeq \frac{1}{2} V \frac{Z_{S=-2}^1}{Z_\Lambda^1 + Z_{\Sigma_0}^1} (Z_{K^+}^1 + Z_{K_0}^1), \tag{6}$$

where the one particle partition functions $Z_{S=-1}^1$ and $Z_{S=-2}^1$ are calculated from Eq. (2) with the corresponding cluster parameters taken from Table 1.

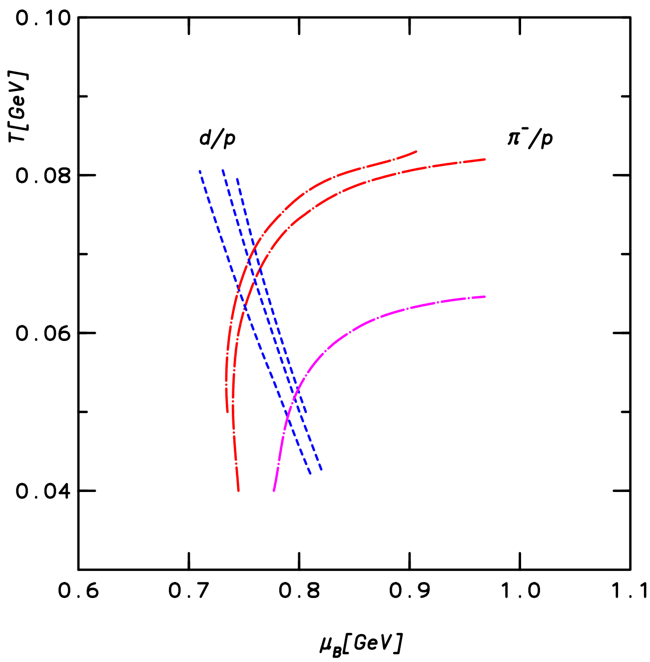

Fig. 2. The lines of constant $d/p = 0.2, 0.26, 0.28$ (counted from left to right) and $\pi^-/p = 0.08, 0.17, 0.193$ (counted from bottom to top) in the (T, μ_B) plane.

To quantify the model predictions for the cluster yields in A–A collisions at SIS energy one needs to specify the values of thermal parameters at chemical freezeout. For a given colliding system the relative production rate (6) depends only on T and μ_B. A very transparent method to pin down these parameters is illustrated in Fig. 2. Considering the experimental data on d/p and π/p ratio in the T–μ_B plane the freezeout point is determined as the crossing of (d/p=const.) and (π/p=const.) lines. [4] For Au+Au collisions at $E_{lab} = 1$ AGeV and in Ni+Ni collisions at $E_{lab} = 1.8$ AGeV the freezeout parameters are ($T \simeq 52$MeV, $\mu_B \simeq 822$ MeV) and ($T \simeq 70$MeV, $\mu_B \simeq 750$ MeV), respectively [52].

Table 2 summarizes the model results for the lower lying $S = -1$ cluster yields in Au+Au and Ni+Ni collisions at $E_{lab} = 1$ AGeV and $E_{lab} = 1.8$ AGeV, respectively. Comparing these results, it is clear that only pK^- and ppK^- yields are sensitive to the collision energy. The heavier states are only weakly affected by an increase in E_{lab} from 1 to 1.8 AGeV. This is rather a surprising result as there is an essential change in the freezeout parameters at these two colliding energies. In addition, due to the different isospin asymmetry in Au and Ni nucleus there is also a shift in the charge chemical potential from $\mu_Q \simeq -19$ MeV in Au+Au to $\mu_Q \simeq 0.0$ MeV in Ni+Ni collisions.

[4] At SIS energy the d/p yields ratio was found [1,42] to be consistent with thermal model predictions when being calculated with the same parameters as all other ratios. This is why the d/p ratio can be used to determine chemical freezeout parameters in heavy ion collisions at SIS energies.

Table 2
Calculated yields for single-K^- clusters for Au+Au at 1 AGeVand Ni+Ni collisions at 1.8 AGeV.

Yield/$\langle\Lambda\rangle$	Au+Au 1 AGeV	Ni+Ni 1.8 AGeV
pK^-	0.399E-02	0.142E-01
ppK^-	0.476E-03	0.129E-02
$pppK^-$	0.309E-02	0.362E-02
$ppnK^-\|_{J^\pi=(1/2)^-}$	0.128E-02	0.128E-02
$ppnK^-\|_{J^\pi=(3/2)^+}$	0.483E-02	0.409E-02
$pnnK^-$	0.713E-02	0.443E-02

The possible presence of kaonic nuclear clusters in heavy ion collisions can in general be verified at the SIS accelerator within the FOPI experimental program dedicated to study strangeness production in Al+Al reactions at 2 AGeV. One way to extend the predictions of the thermal model to higher energy and different colliding systems would be to extrapolate the actual values of the thermal parameters based on the previous systematics. The dependence of freezeout parameters on the system size at SIS energy is, however, at present still not well established. On the other hand, T and μ_B are not direct observables in heavy ion collisions. These thermal parameters can be directly related with observables as shown in Fig. 2. We express the thermal parameters (T, μ_B) through the d/p and π/p ratio and study the dependence of the relative production probability of $\bar{K}$ clusters on the values of these ratios. We consider the change of the cluster production rate with the π/p ratio calculated along a line of constant d/p ratio as shown in Fig. 2.

In Fig. 3 the cluster yields are expressed as a function of the π/p ratio for different values of d/p. The d/p yield ratio is experimentally known to be a decreasing function of the collision energy. In Ni+Ni collisions at $E_{lab} = 1.8$ AGeV the $d/p \simeq 0.28$. In Fig. 3 we illustrate the ppK^- and $pppK^-$ relative yields for three different values of $d/p < 0.28$. Having established the experimental value of d/p and the corresponding π/p ratio in Al+Al collisions at $E_{lab} = 2.0$ AGeV , the expected thermal model results for cluster production probabilities can be obtained from Fig. 4.

The relative yields of double strange clusters are seen in Eq. (6) to be explicitly dependent on the system size in the initial state. In addition, the system size dependence enters through different isospin asymmetry and possible modification of the (T, μ_B) freezeout parameters. In Fig. 4 we illustrate the A–dependence of relative $pppK^-K^-$ cluster yield as a function of π/p ratio along the line of constant $d/p = 0.26$. For fixed π/p ratio, the yields are dropping with decreasing system size. For a sufficiently large π/p value, that is at large $\sqrt{s}$, the canonical suppression is less important and the A–dependence of the production rate is negligible.

The explicit volume dependence seen in the ratio of S=-2 cluster yields per Λ yields arises simply because these particles have different strange quantum numbers. It is clear that normalizing the S=-2 cluster yield to Ξ^- yeild the V dependence is cancelled out.

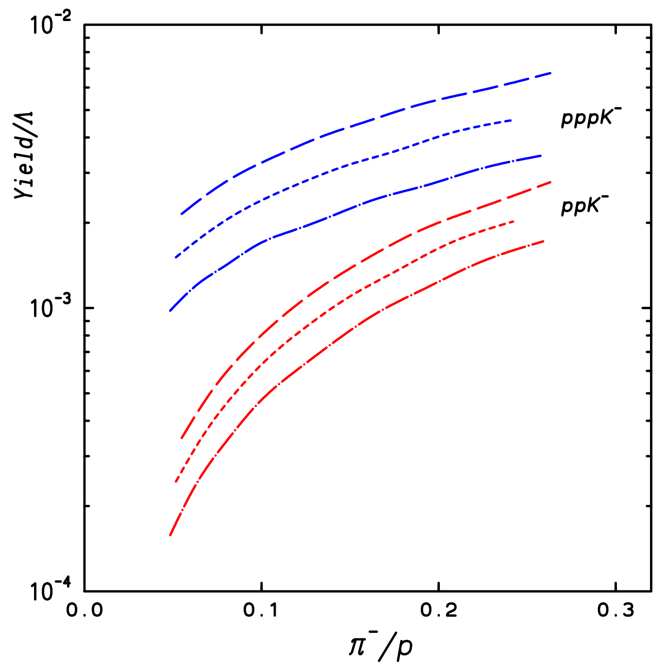

Fig. 3. The yield of ppK^- and $pppK^-$ clusters per Λ yield as a function of the π^-/p ratio. The calculations are done for the fixed d/p ratio. For each cluster the short-dashed line corresponds to $d/p = 0.23$, the lower line to $d/p = 0.20$ and the upper line to $d/p = 0.26$ respectively.

Indeed, in a thermal model the $\langle \Xi^- \rangle / \langle \Lambda \rangle$ ratio is represented by the same expression as $\langle N \rangle^c_{S=-2} / \langle \Lambda \rangle$ in Eq. (6), but with the replacement of $Z^1_{S=-2}$ by $Z^1_{\Xi^-}$. Thus, the ratio

$$\frac{\langle N \rangle^c_{S=-2}}{\langle \Xi^- \rangle} \simeq \frac{Z^1_{S=-2}}{Z^1_{\Xi^-}} \tag{7}$$

is determined by the corresponding ratio of the available thermal particle phase space expressed through the single particle partition functions.

In Fig. 5 the yield of lower lying S=-2 clusters, normalized to the Ξ^- yield, is shown as a function of the π^-/p ratio, for fixed $d/p = 0.26$. These relative yields are not anymore explicitely volume dependent, thus should be the same for all colliding systems. Some small system size dependence could appear, as already indicated, due to a possible modification of chemical freezeout parameters and an isospin asymmetry. However, these effects cannot change the general conclusion that the yield of the double-K^- nuclear bound clusters is by more than three orders of magnitude less abundant than the yield of Ξ^-. Thus, such states are rather unlikely to be identified in heavy ion collisions at the SIS energy [49]. As we have shown in Fig. 1, the yield of double-K^- clusters steeply increases (by about three orders of magnitude) up to $\sqrt{s} \simeq 5$, where it becomes comparable to the single-K^- clusters at SIS energies. This makes their detection feasible at FAIR [50], despite the larger expected combinatorial background. The CBM experiment at FAIR is designed to

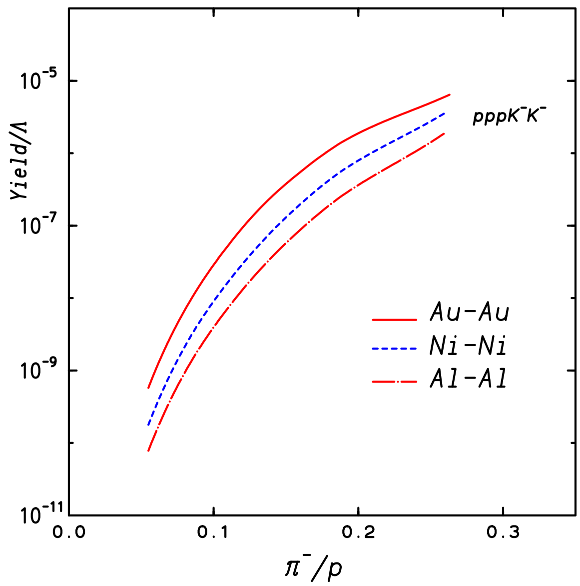

Fig. 4. The yield of $pppK^-K^-$ per Λ yield as a function of π^-/p ratio for Au+Au, Ni+Ni and Al+Al collisions. The calculation are done for fixed $d/p = 0.26$ yield ratio.

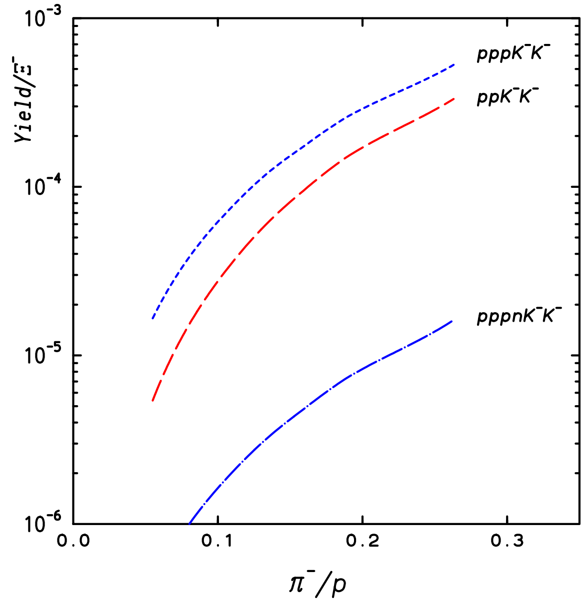

Fig. 5. The yield of ppK^-K^-, $pppK^-K^-$ and $pppnK^-K^-$ clusters per Ξ^- yield as a function of π^-/p ratio. The calculations are done for the fixed $d/p = 0.26$ yields ratio.

study rare probes, including multistrange baryons [51]. By employing a silicon detector to measure displaced vertices and due to the good particle identification and momentum resolution, CBM should be able to detect a variety of kaonic clusters if they are produced at chemical freezeout. The unknown decay channels and branching ratios represent, however, further difficulties for quantitatively assessing the experimental detection possibilities.

4 The scaling relation of cascade production at SIS energy

In the previous section we have emphasized the practical aspect to look experimentally for the yield of the double-K^- relative to Ξ^-. However, the yield of Ξ^- is presently not known in heavy ion collisions in the SIS energy range. To facilitate optimizing future measurements in this respect, we like to emphasize interesting scaling properties of the relative production probabilities of Ξ^- yield normalized to K^+ or Λ yield, predicted within the statistical model. This scaling is illustrated in Fig. 6 where the yield ratio Ξ^-/K^+ is plotted as a function of the K^+/p ratio. The observed linear scaling of Ξ^-/K^+ ratio with K^+/p is independent of the system size. In Fig. 6 we indicate the thermal model predictions of the relative Ξ^-/K^+ yield for different K^+/p ratios obtained at the SIS energy.

The statistical model predicts that such a scaling should be observed if the thermal conditions are such that the canonical suppression effects are dominating the strangeness

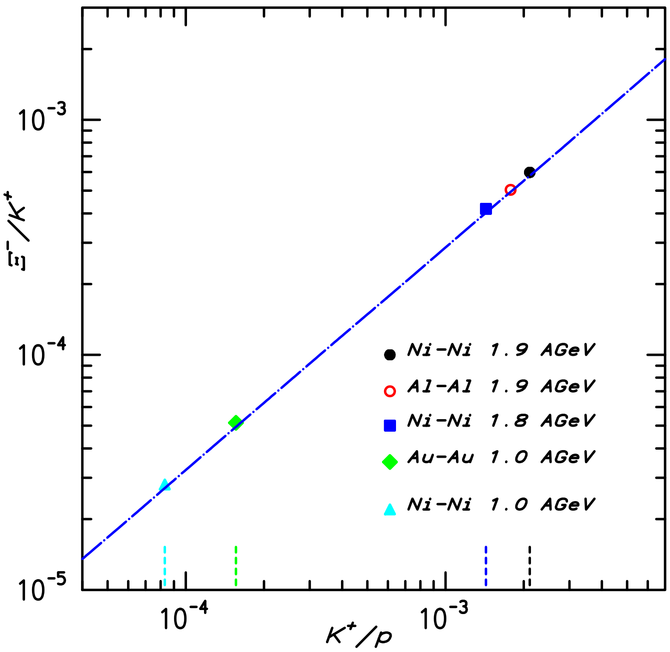

Fig. 6. Scaling properties of Ξ^-/K^+ versus K^+/p ratios. The vertical dashed lines indicate the experimental values of K^+/p ratio from KaoS and FOPI Collaborations [20,21] obtained in Au+Au and Ni+Ni collisions at two different incident energies . The points on the scaling line are the expected results of the Ξ^-/K^+ ratio for the corresponding K^+/p experimental values. The Al+Al point is the prediction of the model.

production rates. The scaling has a more general, dynamical, origin and is a consequence of the production mechanism of kaons, hyperons and cascades at SIS energies.

The Λ and Σ hyperons are produced together with kaons since this is energetically the most favorable way to produce strange hadrons. On the other hand, antikaons are produced below their threshold in binary N–N collisions, thus can be produced only through the strangeness-exchange reaction

$$\pi + Y \rightleftharpoons K^- + N. \tag{8}$$

If the rates for K^- production are equal to those for K^- absorption, the reaction (8) is in a chemical equilibrium. In this case the law-of-mass action is applicable and leads to the following relation between particle yields [52]

$$\frac{[\pi]\cdot[Y]}{[K^-]\cdot[N]} = \kappa_1, \tag{9}$$

with $Y = (\Lambda, \Sigma)$, $[x]$ being the multiplicity of particle x and with κ being the chemical

equilibration constant [52]. On the other hand, the strangeness exchange process

$$K^- + Y \rightleftharpoons \Xi^- + \pi \tag{10}$$

dominates the production of Ξ^- in heavy ion collisions at SIS energies. Thus, here a detailed balance relation implies that

$$\frac{[K^-]\cdot[Y]}{[\Xi^-]\cdot[\pi]} = \kappa_2 \tag{11}$$

From (9) and (11) one concludes that

$$\frac{[\Xi^-]}{[Y]} = \frac{1}{\kappa_1\kappa_2}\frac{[Y]}{[N]}. \tag{12}$$

This relation also holds for the reaction involving direct absorption of Ξ^- on proton, $\Xi^- + p \rightleftharpoons \Lambda + \Lambda$: only if such a process is satisfying a detailed balance relation.

To preserve strangeness conservation the hyperons are produced together with K^+ and K^0 in equal rates, $[Y] = [K^+] + [K^0] \simeq 2[K^+]$, thus from (12) one can also write

$$\frac{[\Xi^-]}{[K^+]} = \kappa \cdot \frac{[K^+]}{[N]}. \tag{13}$$

The above equation explains the linear scaling relation which was obtained within the canonical thermal model, shown in Fig. 6. The relation (12) is valid in the whole SIS energy range for $0.8 < E_{lab} < 2$ A$\cdot$ GeV and is independent of the atomic number of colliding nucleus or centrality in A–A collisions. Thus, Eq. (12) provides a very transparent method to infer the yield of Ξ^- from measurements of the kaon and proton production rates at SIS. For higher collision energies the above scaling is expected to be violated since the processes (8) and (10) are not anymore dominating the production of K^- and Ξ^- hadrons.

Once the proportionality constant κ in Eq. (12) is fixed, the scaling relation (12) provides definite predictions of the Ξ^- production yield for different colliding systems. In general, κ could be calculated within different models, e.g. in dynamical transport models [54], provided that such models are preserving detailed balance relations for the strangeness production. In the statistical model the scaling (12) appears naturally as a consequence of the canonical strangeness conservation and chemical equilibration.

5 Summary and conclusions

The conjecture of the possible existence of deeply bound K^- states has been studied for heavy ion collisions in the context of the statistical thermal model. Based on the analysis of the excitation function of the production yield for various K^- cluster species, we have concluded that the maximum production probabilities of such objects appear at present SIS energies for single-K^- clusters and at the future GSI accelerator in case of double-K^- clusters. This is a direct consequence of a large baryonic density reached in A–A collisions at SIS and of the strong energy dependence of strangeness production at low energies. We have discussed the production yields of single- and double-strange clusters as a function of the system size and of the thermal composition of the collision fireball created at SIS energies. The model predictions on the production probabilities of antikaonic nuclear states relative to the number of hyperons and cascades were quantified.

We have argued that the relative production yield of Ξ^-/K^+ in heavy ion collisions at SIS beam energies should exhibit a linear scaling with the relative yield of K^+/p. Such a scaling appears as a consequence of the strangeness production mechanism at subthreshold energies and the requirement of detailed balance relations for all production reactions. In the statistical thermal model the above scaling appears due to the exact strangeness conservation constraints.

Acknowledgments

We acknowledge stimulating discussions with N. Herrmann, H. Oeschler, J. Wambach and T. Yamazaki. K.R. acknowledges the support of the KBN under the grant 2P03 (06925).

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